

SINGLE-PARTICLE ENERGY LEVELS OF EVEN-EVEN SILICON ISOTOPE

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Abstract

In our research work, single particle energy levels of even-even silicon isotope ^{28}Si are calculated by using the phenomenological Woods-Saxon central potential with spin-orbit interaction. The Gaussian basis wave function is also used in our calculation. The average values of kinetic energy, potential energy and centrifugal energy are calculated. The neutron separation energy and the root-mean-square radius of ^{28}Si are also calculated. Our calculated shell spectra are similar as the prediction of the shell model. Moreover, the calculated last neutron separation energies are in good agreement with the experimental results.

Introduction

There are two basic types of simple nuclear models to investigate the characteristic of the nuclei. Nuclear models generally can be divided into independent particle models (IPM) in which the nucleons in discrete energy states are assumed, to move nearly independently in a common nuclear potential and the strong interaction (collective) models (SIM) in which the nucleons are strongly coupled to each other. The simplest SIM is the liquid drop model. The simplest IPM is also known the Fermi gas model (or) the shell model which is the basis of the semi-empirical mass formula.

The Shell Model

In nuclear physics, the nuclear shell model is a model of the atomic nucleus which uses the Pauli Exclusion Principle to describe the structure of the nucleus in terms of energy levels. The shell model is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. This observation that there are certain magic numbers of nucleons: 2, 8, 20, 28, 50, 82, 126 which

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are more tightly bound than the next higher number, is the origin of the shell model. The nuclear shell model explains the existence of magic numbers.

The energy levels are found by solving the Schrödinger equation for a single nucleon moving in the average potential generated by all other nucleons. Each level may be occupied by a nucleon, or empty. Some levels accommodate several different quantum states with the same energy, they are said to be degenerate.

Some nuclei are bound more tightly than others. This is because two nucleons of the same kind cannot be in the same state. So the lowest energy state of the nucleus is one where nucleons fill all energy levels from the bottom up to some level. The energy levels increase with the orbital angular quantum number l , and s, p, d, f ... symbols are used for $l = 0, 1, 2, 3, \dots$. The value of total angular momentum j and the multiplicity of the state is $2j+1$.

Schrödinger equation and Interaction

In order to find the energy levels of a nucleus, Schrödinger equation is used. The time-independent Schrödinger equation for potential well is as follows.

$$\left[\left(-\frac{\hbar^2}{2M} \nabla^2 \right) + v(r) \right] \Psi(r) = E\Psi(r) \quad (1)$$

The wave function $\Psi(r)$ can be expressed in terms of $R(r) = \Theta(\theta)$ and $\Phi(\phi)$

$$\Psi(r) = R(r)\Theta(\theta)\Phi(\phi) \quad (2)$$

The angular part term,

$$\frac{M^2}{\sin^2 \theta} - \frac{1}{\Theta(\theta)\sin \theta} \left[\cos \theta \frac{d\Theta}{d\theta} + \sin \theta \frac{d^2\Theta}{d\theta^2} \right] = l(l+1) \quad (3)$$

The Radial Part term is,

$$\frac{1}{R(r)} \left[2r \frac{dR(r)}{dr} + r^2 \frac{d^2R(r)}{dr^2} \right] + \frac{2M}{\hbar^2} r^2 (E - V(r)) = l(l+1) \quad (4)$$

So, the radial part of Schrödinger equation is as follows.

$$-\frac{\hbar^2}{2M} \left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right] u(r) + V(r)u(r) = Eu(r) \tag{5}$$

The Gaussian basis wave function is used

$$\Psi(r) = r^{l+1} N \sum_i c_i e^{-\left(\frac{r}{b_i}\right)^2} \tag{6}$$

These linear equations can be solved by using Gauss elimination method which is the fundamental one for solving linear systems. And so norm matrix elements, kinetic energy matrix element and potential energy matrix elements, relative probability density and root-mean-square distance are analytically obtained by using Gaussian standard integral form. By diagonalizing the Hamiltonian matrix, the energy eigen-value is obtained with the help of FORTRAN PROGRAM.

Woods-Saxon Potential

The Dirac equation, which describes the motion of a spin ½ particle, has been used in solving many problems of nuclear and high-energy physics. Within the framework of Dirac equation, p-spin symmetry used to feature deformed nuclei, super deformation, and to establish an effective shell model.

The interactions between nuclei are commonly described by using Woods-Saxon potential that plays a great role in nuclear physics.

The form of the generalized Woods-Saxon potential is as follows.

$$V(r) = -\frac{V_0}{1 + e^{(r-R)/a}} \tag{7}$$

$R = r_0 A^{1/3}$ is the nuclear radius where $r_0 = 1.25$ fm and A is the mass number.

Typical values for the parameters are: $V_0 \approx 50$ MeV, $a \approx 0.5$ fm.

When using the Schrödinger equation to find the energy levels of nucleons subjected to the Woods-Saxon potential, it cannot be solved analytically, and must be treated numerically.

Normalization Constant

To solve the physical quantities of ^{28}Si nuclei, the normalization constant is calculated. The normalized condition is as follows;

$$\int_0^{\infty} \Psi^*(r) \Psi(r) dr = 1 \quad (8)$$

The normalized Gaussian basis wave function is defined as follows;

$$U(r) = A \sum_i c_i e^{-\left(\frac{r}{b_i}\right)^2} r^{l+1} \quad (9)$$

$$A^2 \sum_i \sum_j c_i c_j \int e^{-r^2 \left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)} r^{2(l+1)} dr = 1 \quad (10)$$

By using standard integral, the normalized constant is obtained.

$$A = \frac{1}{\left[\sum_i \sum_j c_i c_j \frac{(2L+1)!!}{2^{l+2}} \frac{1}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)^{l+1}} \left(\frac{\pi}{\frac{1}{b_i^2} + \frac{1}{b_j^2}}\right)^{\frac{1}{2}} \right]^{\frac{1}{2}}} \quad (11)$$

In which normalized matrix element is

$$A_{ij} = \frac{(2l+1)!!}{2^{l+2}} \frac{1}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)^{l+1}} \left(\frac{\pi}{\frac{1}{b_i^2} + \frac{1}{b_j^2}}\right)^{\frac{1}{2}} \quad (12)$$

Kinetic Energy and Centrifugal Potential Energy

Kinetic Energy

The kinetic energy is calculated as follows;

$$\hat{T}(r)U(r) = \frac{\hbar^2}{2\mu} \left[\frac{d^2}{dr^2} U(r) \right] \quad (13)$$

$$\hat{T}(r)U(r) = \frac{\hbar^2}{2\mu} A \sum_i c_i e^{-\left(\frac{r}{b_i}\right)^2} \left[l(l+1)r^{l-1} - \frac{2}{b_i^2}(2l+3)r^{l+1} + \frac{4}{b_i^4}r^{l+3} \right] \quad (14)$$

Kinetic energy matrix element is as follows

$$T_{ij} = \frac{\hbar^2}{2\mu} A e^{-\left(\frac{r}{b_i}\right)^2} \left[l(l+1)r^{l-1} - \frac{2}{b_i^2}(2l+3)r^{l+1} + \frac{4}{b_i^4}r^{l+3} \right] \quad (15)$$

Centrifugal Potential Energy

The centrifugal potential energy is follows

$$\hat{F}(r)U(r) = \frac{\hbar^2}{2\mu} \left[\frac{l(l+1)}{r^2} U(r) \right] \quad (16)$$

By using wave function, the centrifugal potential energy is described as follows;

$$\hat{F}(r)U(r) = \frac{\hbar^2}{2\mu} \left[\frac{l(l+1)}{r^2} A \sum_i c_i e^{-\left(\frac{r}{b_i}\right)^2} r^{l+1} \right] \quad (17)$$

The centrifugal potential matrix element is as follows;

$$F_{ij} = \frac{\hbar^2}{2\mu} \left[\frac{l(l+1)}{r^2} A e^{-\left(\frac{r}{b_i}\right)^2} r^{l+1} \right] \quad (18)$$

Potential Energy

In our calculation the phenomenological Woods-Saxon Potential is used. The form of the generalized Wood-Saxon potential is as follows.

$$V(r) = \frac{-V_0}{1 + e^{(r-R_0)/a}} \quad (19)$$

In this equation $R_0 = r_0 A^{1/3}$ is the nuclear radius and A is the diffuse parameter. r_0 is the radii distance from the center. The potential strength depends upon the number of proton and neutron. It is represent by,

$$V_0 = \left[50 - 32 \frac{N-Z}{A} \right] \text{ and the nuclear density, } \rho_{(r)} = \frac{1}{1 + e^{r-R/a}}.$$

Woods-Saxon potential including spin-orbit interaction is

$$V(r) = V_{w-s}(r) + V_{ls}(r) \quad (20)$$

The spin-orbit interaction is described as follows.

$$V_{ls}(r) = V_{so} \left(\frac{\hbar}{Mc} \right)^2 (l.s) \frac{1}{r} \frac{d\rho}{dr}$$

The total angular momentum is $\vec{J} = \vec{l} + \vec{s}$ and the spin of a nucleon is $\frac{1}{2} \hbar$.

$$\text{For } j = l + \frac{1}{2}, l.s = \frac{1}{2}l$$

$$\therefore V(r) = V_{w-s}(r) + V_{ls}(r) \quad (21)$$

$$\begin{aligned} V(r) &= -V_0 \rho(r) + V_{ls}(r) \\ &= \frac{-V_0}{1 + e^{r-R/a}} + V_{so} \left(\frac{\hbar}{Mc} \right)^2 \left(\frac{1}{2}l \right) \left[\frac{1}{r} \frac{e^{r-R/a}}{1 + e^{r-R/a}} \frac{1}{a} \right] \end{aligned}$$

$$\text{For } j = l - \frac{1}{2}, l.s = -\frac{1}{2}(l+1)$$

$$\therefore V(r) = \frac{-V_0}{1 + e^{r-R/a}} - V_{so} \left(\frac{\hbar}{Mc} \right)^2 \left(\frac{1}{2}(l+1) \right) \left[\frac{1}{r} \frac{e^{r-R/a}}{1 + e^{r-R/a}} \frac{1}{a} \right] \quad (22)$$

Probability Density and Root-Mean-Square Distance

To analyze the structure of the nuclei, the probability density and root-mean-square radius are calculated.

Probability Density

The probability density is follows

$$\int_{r_1}^{r_2} P(x, t) dx = \int_{r_1}^{r_2} \psi^*(x, t) \psi(x, t) dx \tag{23}$$

By applying the Gaussian wave function, the probability density is calculated.

$$\text{Probability density} = A^2 \sum_i c_i \sum_j c_j \frac{(2l+1)!!}{2^{l+2}} \frac{1}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)^{l+1}} \left(\frac{\pi}{\frac{1}{b_i^2} + \frac{1}{b_j^2}}\right)^{1/2} \tag{24}$$

Root-Mean Square Distance

The root-mean square distance of a particle is as follows.

$$\langle r^2 \rangle = \int \psi^* r^2 \psi dr \tag{25}$$

By using normalized wave function, the root-mean square distance is described as follows.

$$\langle r^2 \rangle = A^2 \sum_i c_i \sum_j c_j \frac{(2l+3)(2l+3-2)!!}{2^{l+2} \times 2} \frac{1}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)^{l+1}} \frac{1}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)} \left(\frac{\pi}{\frac{1}{b_i^2} + \frac{1}{b_j^2}}\right)^{1/2} \tag{26}$$

Results and Discussions

The single-particle energy levels of ²⁸Si are calculated by using Gaussian basic wave function and the phenomenological Woods-Saxon potential including spin orbit interaction. The root-mean square radius and the last neutron separation energy of ²⁸Si are also investigated.

The calculated results of energy levels of ²⁸Si are shown in Table (1) and the corresponding energy shell levels are shown in Fig. (1). In this Fig. (1), the innermost level 1s_{1/2} among the calculated other levels has the highest binding energy and it has gradually decrease to 1f_{7/2} levels away from the ²⁸Si nucleus. The higher energy levels of 1s_{1/2} are split into two energy levels for same orbital angular momentum. The neutron separation energy of ²⁸Si is also calculated and it is 8.447744MeV and it is nearly equal to experimental results.

Finally, the single-particle energy levels of even-even silicon isotope ^{28}Si and ^{30}Si are calculated and are compared in Table (2).

Conclusion

In our research work, the ground state energy of the most abundance stable ^{28}Si are calculated by using phenomenological Woods-Saxon potential including spin-orbital interaction and the strength of the potential is 50MeV, $r_0 = 1.25\text{fm}$ and diffuse parameter $a=0.53$. It is found that Woods-Saxon potential provides a model for the properties of bound-state and continuum single-particle wave functions. Our calculated results are in good agreement with the experimental results.

Table 1: The Physical Quantities of ^{28}Si

Single-Particle State of ^{28}Si	Total Energy (MeV)	Average Root-Mean Square Radius (fm)	Normalized Constant
$1s_{\frac{1}{2}}$	-36.243832	2.285874	32.9676497
$1p_{\frac{3}{2}}$	-25.328756	2.843373	21.2907179
$1p_{\frac{1}{2}}$	-22.553915	2.779176	7.974858
$1d_{\frac{5}{2}}$	-13.679509	3.329208	12.1807297
$1d_{\frac{3}{2}}$	-8.0593294	3.331005	13.880731
$1f_{\frac{7}{2}}$	-1.8267510	3.944157	2.687783

Table 2: Energy levels of Even-Even Silicon Isotope

Energy States		Total Energy (MeV)	
		$^{28}_{14}\text{Si}$	$^{30}_{14}\text{Si}$
s-state	s1/2	-36.243832	-36.76636
p-state	p3/2	-25.328756	-26.12143
	p1/2	-22.553915	-23.52642
d-state	d5/2	-13.679509	-14.63793
	d3/2	-8.0593294	-9.303161
f-state	f7/2	-1.8267510	-2.730027

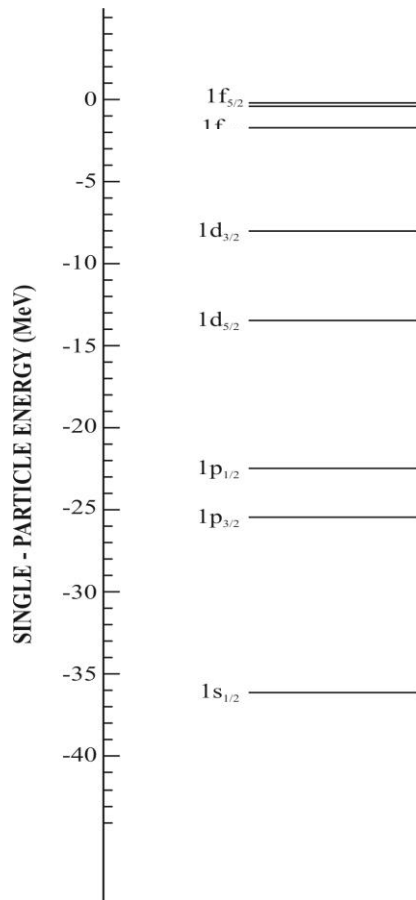


Figure 1: Neutron single-particle energy levels of ^{28}Si

Acknowledgements

I would like to express our gratitude and sincere thanks to Rector Dr Thura Oo, Monywa University for their permission to do this work. I wish to thank Dr Shwe Zin Aung, Professor and Head, Department of Physics, Monywa University for her precious advice, discussions and guidance.

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